Constant mean-curvature surfaces under complex constraints

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To minimize their energy, soap films form surfaces whose mean curvature is given by the pressure difference Δp between the two sides of the film. The nature and shape of these surfaces generally depend on the boundary conditions and other imposed constraints, such as a volume constraint. In the absence of volume stresses $(\Delta p = 0)$, the mean curvature of the surface is zero. Such shapes are known as minimal surfaces that have been extensively studied both by physicists and mathematicians, for their fascinating properties and their applications in biophysics, foam physics, material science or architecture [1]. When submitted to a pressure difference, the mean curvature is constant but different from zero, leading to the more general mathematical class of constant-mean-curvature surfaces (Delauney surfaces). Such surfaces (catenoids, unduloïds or nodoïds) have been widely investigated mathematically, but are not well known from an experimental point of view and many challenging questions remain open concerning their behavior under complex constraints. Membranes and soap films are excellent model systems to advance on these questions.

We have recently developed experimental, theoretical and numerical tools to study the complex behavior of two bubbles in contact [2]. We have also developed perturbative theories to describe non-axisymmetric minimal surfaces [3]. This work opens up a wide range of possibilities for better understanding the behavior of bubbles or drops in environments with complex constraints.

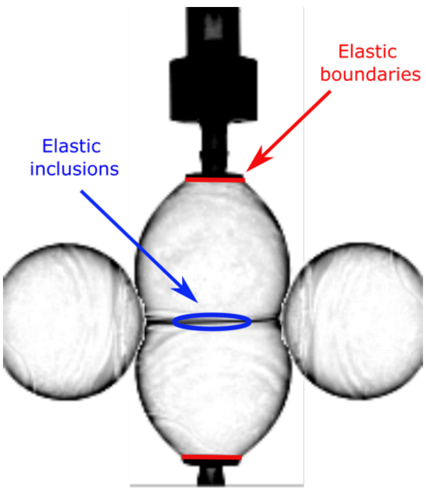

Figure 1: Schematic view of two interacting bubbles in complex environments: many bubbles; flexible boundaries; elastic insertions.

The first part of this PhD will aim to develop a perturbative theory to describe non-axisymmetric surfaces with constant non-zero mean curvature, and to test it experimentally and numerically. Particular attention will be paid to studying the stability of these surfaces, and also to the properties linked to the breakage of axi-symmetry, such as the existence of a torsional torque. In the second part, the aim will be to understand the interactions between two bubbles in a complex environment, focusing in particular on the following cases: (i) the presence of other bubbles (Figure 1); (ii) flexible boundary conditions; (iii) the presence of flexible inclusions in the soap film. In all cases, we will combine experiments, simulations (Surface Evolver) and theory.

The candidate will join the M3 of ICS and work in collaboration with the MIM team in their combined efforts to understand the physics of membranes and thin films. She/he should have a strong background in general physics. The project is mainly experimental, with aspects of modelling and numerical simulations.

[1] The science of soap films and soap bubbles, by Isenberg (1978).
[2] F. Walzel et al., Soft Matter, 10.1039/D4SM00919C (2024).
[3] F. Walzel et al., Physical Review E, 106 (2022).